Scaling behaviour in the growth of companies

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- **1996, Nature**
- **Link to paper**

Central Findings:

- For companies with similar initial sales, the distribution of annual (logarithmic) growth rates has an exponential form.
    - Furthermore, the spread in the distribution of rates decreases with increasing sales **as a power law over seven orders of magnitude.**
- The above finding applies to firms of various kinds
    - They conclude that the growth of a company may be tied more closely to the organizational structure of the company as opposed to production-related factors.

Intro/Lit:

- The simplest model of corporate growth was proposed by Gibrat. The basic assumptions are that the *rate of company growth is*:
    - Independent of company size (law of proportionate effect)
    - Uncorrelated in time
- Mathematically this is…

    formalized by the following random multiplicative process: $S_{t+\Delta t} = S_t(1+\varepsilon_t)$, where $S_{t+\Delta t}$ and S_t are the sales of the company at time $t+\Delta t$ and t respectively, and ε_t is an uncorrelated random number with mean close to zero and standard deviation much smaller than one. Hence $\log S_t$ follows a simple random walk so that firm sizes are log–normally distributed. Also, for sufficiently large time intervals $T \gg \Delta t$, the growth rates S_{t+T}/S_t are log–normally distributed.

- It is well known that these assumptions are rejected empirically, yet many people use it as a benchmark for lack of a better option

Methods/Approach:

- Study all US manufacturing publicly traded companies between '75-'91
    - All data taken from Compustat database
    - All data adjusted to 1987 dollars via GNP price deflator
- Define a firm's annual growth rate as:

$$R \equiv {S_1}/{S_0}$$

    - where S_0 and S_1 are its sales in two consecutive years

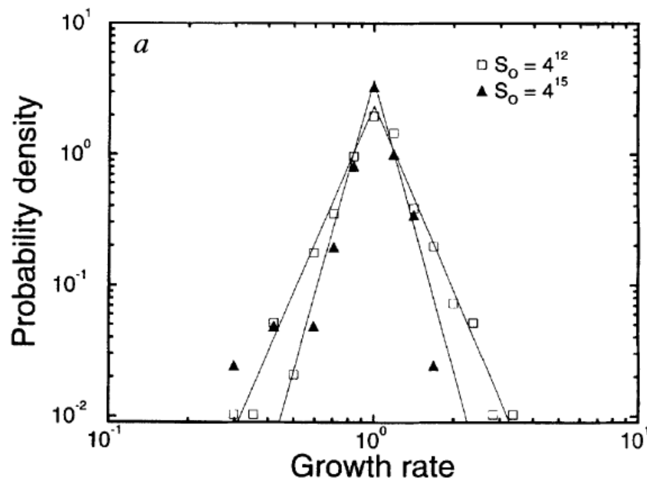
    It is customary to study company growth on logarithmic scales, so we define $r \equiv \ln(S_1/S_0)$ and $s_0 \equiv \ln S_0$ and calculate the conditional distribution $p(r \mid s_0)$ of growth rates r with a given initial sales value s_0.

Growth Rate:

- Analyzing the data, they find that it fits well to a non-Gaussian, simple "tent-shaped" form

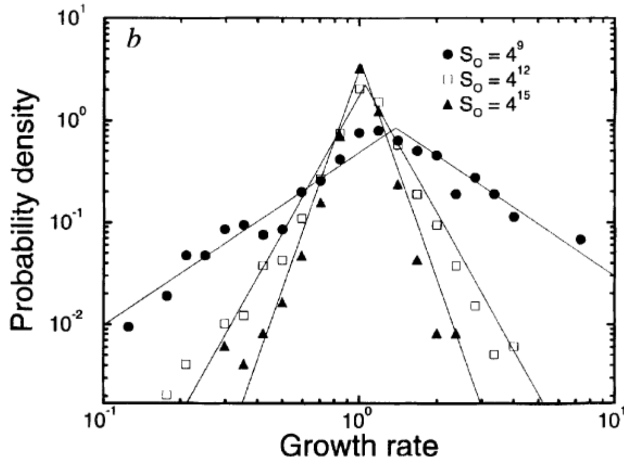
$$p(r \mid s_0) = \frac{1}{\sqrt{2}\sigma(s_0)} \exp\left(- \frac{\sqrt{2}|r - \bar{r}(s_0)|}{\sigma(s_0)} \right)$$

- We see (fig. below) the probability density $p(r|s_0)$ of the growh rate $r \equiv \ln\left({}^{S_1}/_{S_0} \right)$ from '90 to '91
  - Data for 2 different **bins** of *initial states* with sizes that increase by a power of 4
    - for example, $4^{11.5} < S_0 < 4^{12.5}$ (squares)



  - The solid lines are fits to the equation above using the mean $\bar{r}(s_0)$ and standard deviation $\sigma(s_0)$ calculated from the data.

- They then calculate the average over all 16 one-year periods and bin it into three different bins this time…



- The solid lines once again are fitted to the above equation and fit quite well

- We see in the figure above that the $\sigma(s_0)$ decreases as s_0 increases.
- The below law (equation) approximates $\sigma(s_0)$ quite well over more than seven orders of magnitude
    - From 10^4 dollars to more than 10^{11} dollars

$$\sigma(s_0) = a \exp(-\beta s_0) = aS_0^{-\beta}$$

where $a \approx 6.66$ and $\beta = 0.15 \pm 0.03$

Other indicators of growth:

- A similar analysis was conducted for the number of employees
    - *Comparison of sales and employees seen below*
- Data show a linear relationship which spans five orders of magnitude
    - From firms with 10 employees to firms with almost 10^6 employees
    - Slope $\beta = 0.16 \pm 0.03$ is the same, within error bars, as that found for sales
- **They also find that the above equation accurately describes three additional indicators of company growth**
    - Cost of goods sold (exponent $\beta = 0.16 \pm 0.03$)
    - Assets (exponent $\beta = 0.17 \pm 0.04$
    - Property, plant and equipment (exponent $\beta = 0.18 \pm 0.03$)

- **Additionally, this law maps to a diverse set of firms which range in size as well as the products they manufacture**

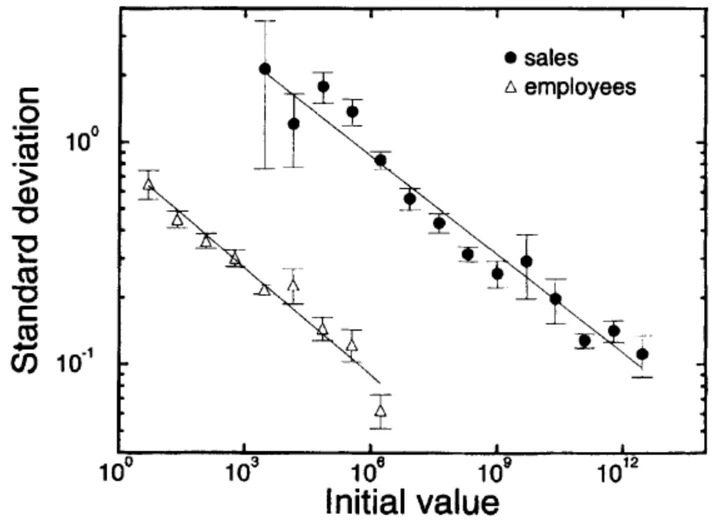


FIG. 2 Standard deviation of the one-year growth rates of the sales (circles) and of the one-year growth rates of the number of employees (triangles) as a function of the initial values. The solid lines are least-square fits to the data with slopes $\beta = 0.15 \pm 0.03$ for the sales and $\beta = 0.16 \pm 0.03$ for the number of employees. We also show error bars of one standard deviation about each data point. These error bars appear asymmetric as the ordinate is a log scale.

Conventional Perspective Statistical Physics Perspective:

- Conventional theory does not suggest that different types of companies would grow at the same rate
- The findings above are more reminiscent of a universal concept found in statistical physics – where different systems can be characterized by the same fundamental laws independent of the underlying details
- A common test is to transform the different variables to the same scale to show the scale-variant behavior (see below)

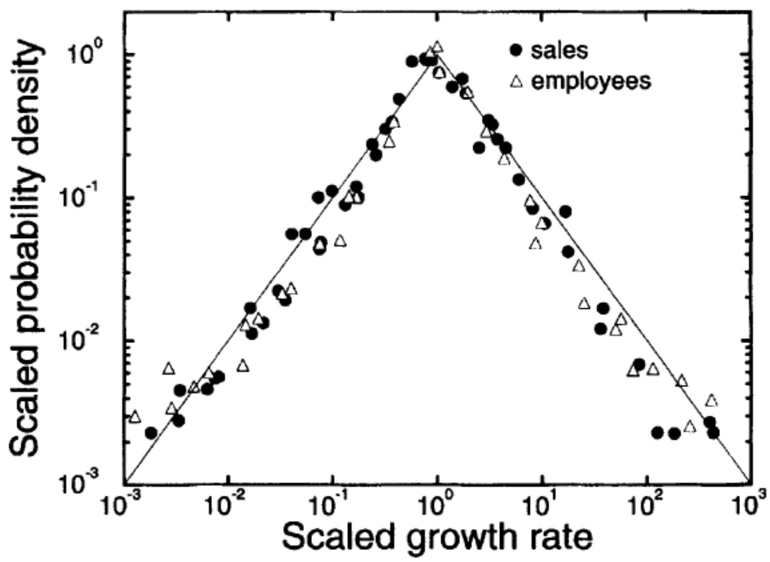


FIG. 3 Scaled probability density $p_{scal} \equiv 2^{1/2}\sigma(s_0)p(r \mid s_0)$ as a function of the scaled growth rate $r_{scal} \equiv 2^{1/2}[r - \bar{r}(s_0)]/\sigma(s_0)$ of sales (circles). The values were rescaled using the measured values of $\bar{r}(s_0)$ and $\sigma(s_0)$. Also we show (triangles) the analogous scaled quantities for the number of employees. All the data collapse upon the universal curve $p_{scal} = \exp(-\mid r_{scal} \mid)$ (solid line) as predicted by equations (1) and (2).

A very brief additional analysis looking at the hierarchy of a company is squeezed into the end of this paper but I am leaving this out as the above is representative of the central results, based on real-world data, while this additional analysis is based on largely on mathematical formulation and theoretical assumptions.